AT
FH
LB
KW
AW
NB
YR
JG

Name: ____________________

Class: 12MT2__ or 12MTX__

Teacher: ____________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2007 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS

Time allowed - 3 HOURS
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- *Each question is to be commenced on a new page clearly marked* Question 1, Question 2, etc on the top of the page. **.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used. Standard Integral Tables are provided
- Your solutions will be collected in one bundle stapled in the top left corner.

Please arrange them in order, Q1 to 10.

*****Each page must show your name and your class. *****

QUESTION 1 MARKS

(a) Evaluate to 2 decimal places $\frac{2\cdot 34\times 4\cdot 654}{2\cdot 786-1\cdot 658}$ **2**

(b) Express $\sqrt{27}+\sqrt{48}$ in the form $a\sqrt{3}$ where a is an integer **2**

(c) Solve for x, $|x-4|<6$ **2**

(d) Find the exact value of $\tan\frac{2\pi}{3}$ **2**

(e) Factorise $1+8x^3$ **2**

(f) Solve $2x-\frac{3x+4}{5}=9$ **2**

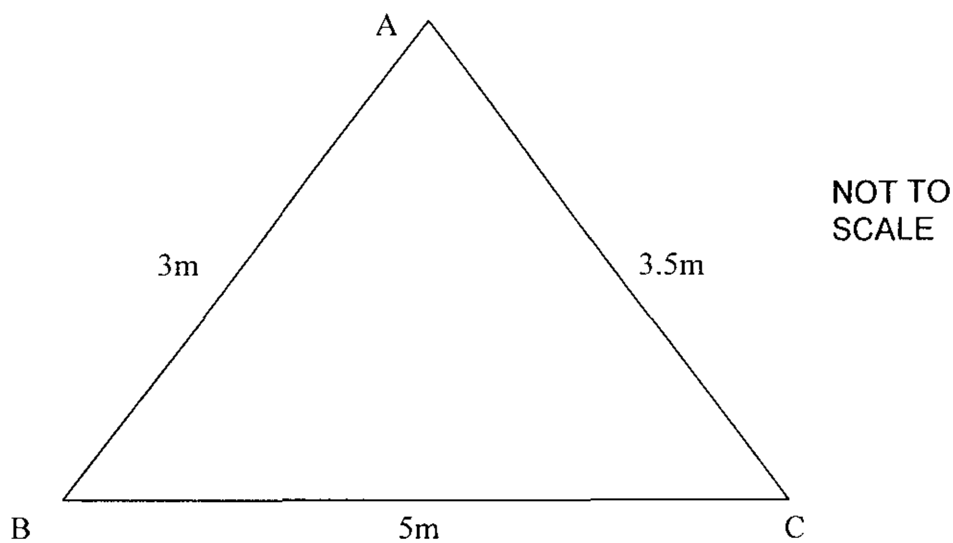
QUESTION 2 *(Start a new page)* **MARKS**

(a) Differentiate with respect to x

(i) $\dfrac{x}{\ln x}$ **2**

(ii) $(1+\tan x)^5$ **2**

(b)



In the diagram above AB= 3m, BC= 5m and AC=3.5m. Find the size of the largest angle, correct to the nearest minute. **3**

(c) (i) Find in exact form $\displaystyle\int_0^2 \frac{6x^2}{1+x^3}\,dx$ **3**

(ii) Find $\displaystyle\int\left(\frac{1}{3x^2}+e^{3x}\right)dx$ **2**

QUESTION 3 *(Start a new page)* **MARKS**

(a) The co-ordinates of points A, B and C are $A(1,4)$, $B(-2,1)$ and $C(2,-2)$.

(i) Plot these points on a number plane **1**

(ii) Show that the equation of the line AB is $x - y + 3 = 0$ **2**

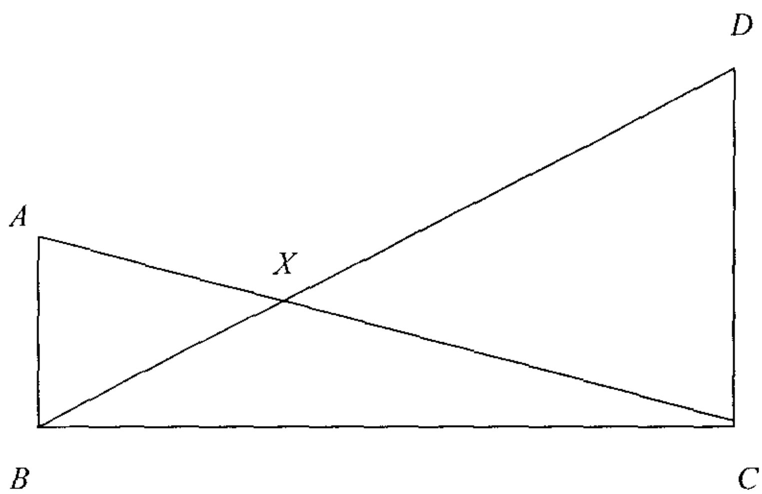
(iii) Find the perpendicular distance from C to AB **2**

(iv) Find the distance AB (leave answer in simplest surd form) **1**

(v) Hence, find the area of ΔABC, correct to one decimal place. **1**

(b) Show that log 2, log 4, log 8, log 16 form an arithmetic sequence **1**

(c) In the diagram AB is parallel to DC and $AB : DC = 2 : 3$

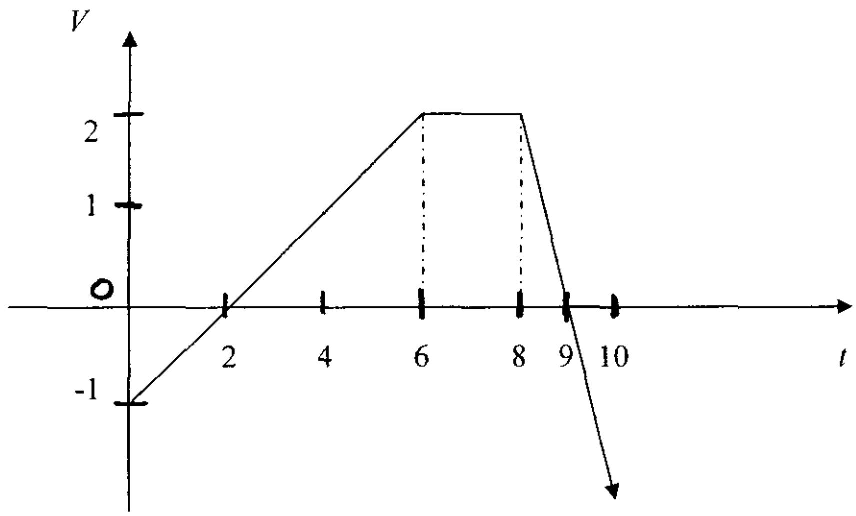


(i) Show that $\Delta ABX \,|||\, \Delta CDX$ **2**

(ii) Show that $9BX^2 = 4XD^2$ **2**

QUESTION 4 *(Start a new page)* **MARKS**

(a) A particle moves in a straight line with a velocity of Vms^{-1}.
The graph of $V = f(t)$ is illustrated on the diagram below.



(i) When is the particle at rest? **1**

(ii) When does the particle first return to its starting point? **1**

(iii) Find the total distance travelled by the particle in the first 9 seconds. **1**

(b) The geometric series $1 - x + x^2 - ....$ has a limiting sum of 4. **2**
Find the value of x.

(c) Find the co-ordinates of the focus of the parabola $6y = x^2 - 2x - 11$ **3**

(d) The quadratic equation $2x^2 - 3x - 6 = 0$ has roots α *and* β.
Without solving the quadratic equation, find the value of:

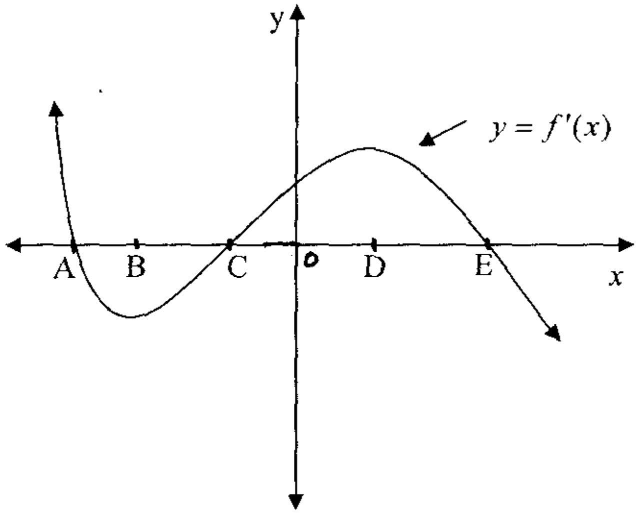
(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

QUESTION 5 *(Start a new page)* **MARKS**

(a)



The above diagram shows a sketch of the function $y = f'(x)$. **2**
Sketch the function $y = f(x)$, indicating any stationary points.

(b) A function $f(x)$ is defined by $f(x) = x^3 - 3x^2 - 9x$.

(i) Find the turning points for the curve $y = f(x)$ and determine their nature. **3**

(ii) Find any point(s) of inflexion. **2**

(iii) Sketch the graph of $y = f(x)$ showing the turning points and point of inflexion. **1**

(iv) Find the values of x for which both $f'(x) < 0$ *and* $f''(x) > 0$. **2**

(c) Solve $2\sin^2\theta - 1 = 0$ for $0 \le \theta \le 2\pi$ **2**

QUESTION 6 *(Start a new page)* **MARKS**

(a) If $y = A\cos 4x + B\sin 4x$, show that $\dfrac{d^2y}{dx^2} + 16y = 0$ **2**

(b) The rate in litres per day at which a faulty tank is losing water is given by

$$R = 20 - \frac{20}{t+1}.$$

(i) At what rate is the tank losing water after 4 days? **1**

(ii) What value does R approach as t becomes very large? **1**

(iii) Calculate the amount of water lost from the tank, in litres, over the first 9 days. (Leave your answer in exact form.) **2**

(c) The population P of a certain bacteria is falling according to the formula:

$P = 3000e^{-kt}$, where t is in days .

(i) Show that $\dfrac{dP}{dt} = -kP$ **1**

(ii) If it takes 4 days for the number of bacteria to fall to 2000, what is the value of k (correct to 4 decimal places)? **2**

(iii) Find the rate of change of the population after one week. (Answer correct to 1 decimal place) **1**

(iv) How long will it take for the number of bacteria to fall to 10% of the initial number? (Answer correct to 1 decimal place) **2**

QUESTION 7 *(Start a new page)* **MARKS**

(a) Use Simpson's Rule with five function values to find an approximation for the value of $\int_0^1 10^x \, dx$. Give your answer correct to three decimal places. **2**

(b) (i) Show that $x = \frac{\pi}{8}$ is a solution of $\sin 2x = \cos 2x$ **1**

(ii) On the same set of axes, sketch the graphs of the functions $y = \sin 2x$ and $y = \cos 2x$ for $-\pi \leq x \leq \pi$. **3**

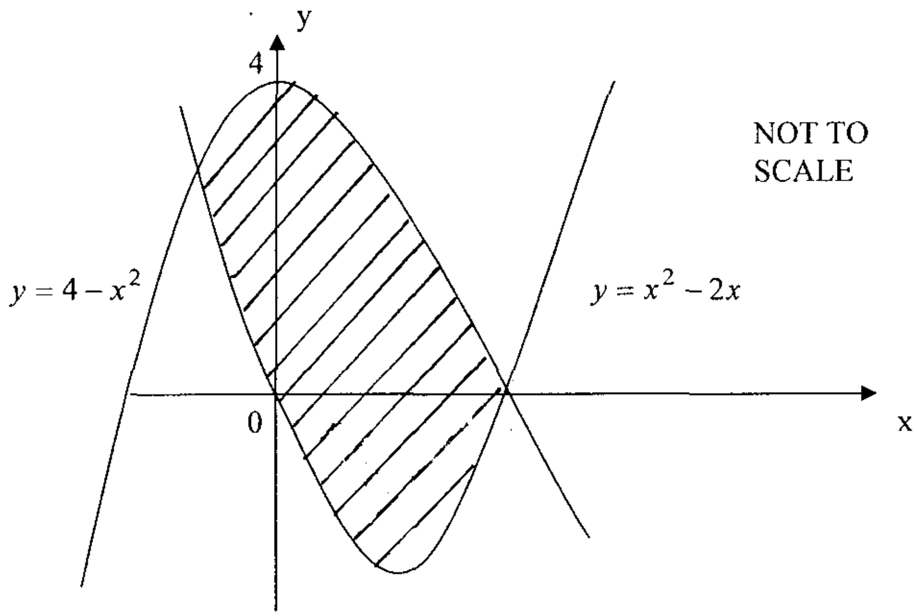
(iii) Hence, find graphically the number of solutions of $\tan 2x = 1 \ \textit{for} \ -\frac{\pi}{2} < x < \frac{\pi}{2}$. **1**

(iv) Use part (ii) to solve $\tan 2x \leq 1 \ \textit{for} \ -\frac{\pi}{2} < x < \frac{\pi}{2}$ **1**

(c) The region enclosed between the curve $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, the x axis, and the lines $x = 4 \ \textit{and} \ x = 11$ is rotated about the x axis. Calculate the exact volume of the solid generated. **4**

QUESTION 8 *(Start a new page)* **MARKS**

(a) The graphs of the functions $y = 4 - x^2$ and $y = x^2 - 2x$.



(i) Describe, using inequalities, the shaded region. **2**

(ii) By solving simultaneously, show that the points of intersection are at $x = -1$ *and* $x = 2$. **1**

(iii) Calculate the area of the shaded region. **2**

(b) A particle moves along a straight line so that its distance x, in metres from a fixed point O is given by $x = t + \cos t$, where t is in the time measured in seconds.

(i) Where is the particle initially? **1**

(ii) When, and where, does the particle first come to rest? **3**

(iii) When does the particle next come to rest? **1**

(iv) Find the acceleration of the particle when $t = \dfrac{\pi}{6}$ seconds. **2**

QUESTION 9 *(Start a new page)* **MARKS**

(a) Simplify $\frac{\operatorname{cosec} A \sec A}{\tan A}$, expressing your answer in its simplest possible form. **1**

(b) Farmer Hay has hired a driller to drill a borehole to enable her to have access to the underground water on her property. The driller quotes a price of \$260 for the first 3 metres drilled, \$280 for the next 2 metres, \$300 for the next 2 metres and so on. The price increases by the same amount for each successive 2 metres of borehole drilled.

(i) Show that the cost of drilling the portion from a depth of 25 metres to 27 metres is \$500. **2**

(ii) Calculate the total cost of drilling to a depth of 27 metres. **1**

(iii) The cost of drilling the borehole to reach water was \$12 500. Find the total depth drilled to give access to the water. **2**

(c) Bill and Ben decide to build a yacht to sail around the world. They approach a bank to see how much they can borrow. The bank manager tells them they can borrow \$P. They will receive an introductory rate of 6%p.a. for the first 3 months. The loan needs to be initially repaid in equal monthly repayments of \$3500 over 5 years and interest is charged monthly before each repayment.

Let $\$A_n$ be the amount owing by Bill and Ben at the end of the nth repayment.

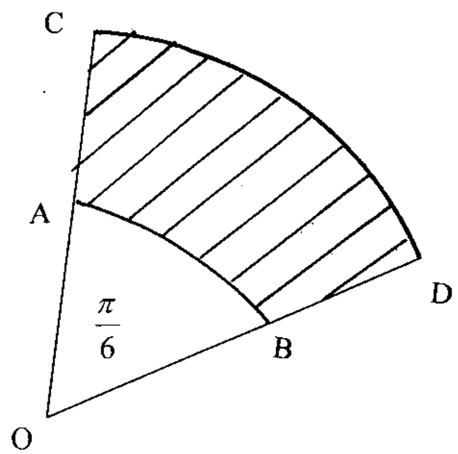
(i) Find an expression for A_1 **1**

(ii) Show that $A_3 = P(1.005)^3 - 3500(1 + 1.005 + 1.005^2)$ **2**

(iii) At the end of the first three months Bill and Ben decide to fix the interest rate at 9% p.a. for the remainder of the loan. They make monthly repayments of \$4200 for the next 4.75 years. Calculate the value of $\$P$. **3**

QUESTION 10 *(Start a new page)* **MARKS**

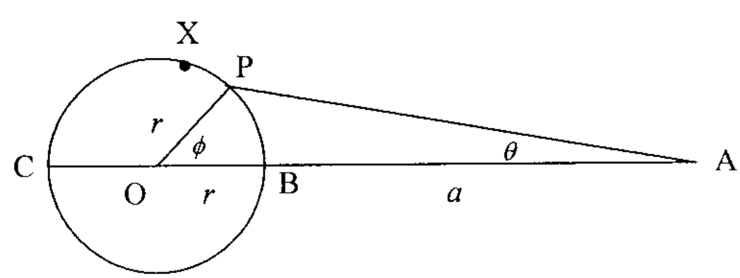
(a)



AB and CD are arcs of concentric circles with centre O. **2**

$\angle AOB = \frac{\pi}{6}$ *radians*. OA = 2 centimetres. The shaded section has area $5\pi\, cm^2$. Calculate the length of AC.

(b) In the diagram below, the fixed points A, B, O, C, lie on a straight line. The points B, P, X, C are on the circumference of a circular pond, centre at O, radius r. AX is a tangent to the circle at X, and P lies on the arc BX.



The distance AB is a, $\theta = \angle PAB$ and $\phi = \angle POB$. The interval AP lies outside the pond. A boy, initially at A, is to walk in a straight line to P, at a constant speed $v_1 > 0$. He is then to run at a constant speed v_2, where $v_2 > v_1$, along the shorter of the two arcs from P to C.

Question 10 (b) continued on the next page.....

(i) Show that the total time T for the journey is **3**

$$T = \frac{\sqrt{r^2 + (r+a)^2 - 2r(r+a)\cos\phi}}{v_1} + \frac{r(\pi - \phi)}{v_2}$$

(ii) Find $\frac{dT}{d\phi}$ **1**

(iii) By using parts (i), (ii) and the sine rule show that **2**

$$\frac{dT}{d\phi} = \frac{r+a}{v_1}\left[\sin\theta - \frac{rv_1}{(r+a)v_2}\right]$$

(iv) Given that $\angle AXO = 90^0$ show that $\frac{dT}{d\phi} < 0$ if P is at B, **2**

and $\frac{dT}{d\phi} > 0$ if P is at X.

(v) Find, in terms of $\sin\theta$, the position of P which minimizes **2**

the time T.

END OF EXAM

Marking Guidelines
AT

Trial Paper 2007 : Mathematics : Solutions

Q1. a) 9.654574468 (1)
9.65 (1)

b) $3\sqrt{3} + 4\sqrt{3}$ (1)
$= 7\sqrt{3}$ (1) No marks for CFM

c) $-6 < x - 4 < 6$ (1)
$-2 < x < 10$ (1)

d) $\tan \frac{2\pi}{3} = -\tan \frac{\pi}{3}$ (1) for second quadrant (negative)
$= -\sqrt{3}$ (1) (aw 1 for $\sqrt{3}$)

e) $\underbrace{(1+2x)}_{1}\underbrace{(1 - 2x + 4x^2)}_{1}$

f) $10x - 3x - 4 = 45$ (1)
$7x = 49$
$x = 7$ (1)

[aw 1 for CFM
ie $10x - 3x + 4 = 45$
$x = \frac{41}{7} = 5.86 = 5\frac{6}{7}$]

Q2. a) (i) $\frac{d}{dx} = \frac{\ln x \times 1 - x \times \frac{1}{x}}{(\ln x)^2}$ (1)
$= \frac{\ln x - 1}{(\ln x)^2}$ (1)

[$\ln x^2 \neq (\ln x)^2$
$\ln^2 x$] - 1 mark

(ii) $\frac{d}{dx} = \underbrace{5(1 + \tan x)^4}_{1} \times \underbrace{\sec^2 x}_{1}$ $= 5\sec^2 x\,(1 + \tan x)^4$

b) $\cos A = \frac{3^2 + 3.5^2 - 5^2}{2 \times 3 \times 3.5}$ (1) incorrect formula = 0 marks
incorrect sign
$(100^\circ\ 17'\ 11.6'')$ (1) $79^\circ 43'$ (- 1 mark)
$A = 100^\circ\ 17'$ (1)

* cos rule after using sine rule
⇒ ambiguous case ⇒ 0 marks

Q2 cont'd (c) (i) $\int_0^2 \frac{6x^2}{1+x^3}\,dx = 2\int_0^2 \frac{3x^2}{1+x^3}\,dx$

$= 2\left[\ln(1+x^3)\right]_0^2$ ①

$= 2\ln(1+2^3) - 2\ln(1+0^3)$ ①

$= 2\ln 9 - 2\ln 1$

$= 2\ln 9$ ① accept either

$= \ln 81$

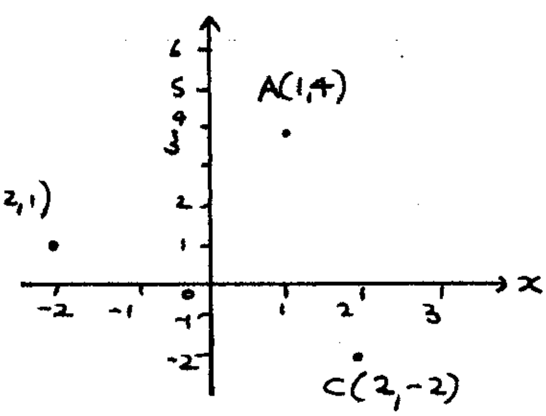
(ii) $\int \frac{1}{3x^2} + e^{3x}\,dx = \int \frac{1}{3}x^{-2} + e^{3x}\,dx$

$= \underbrace{-\frac{1}{3}x^{-1}}_{①} + \underbrace{\frac{e^{3x}}{3}}_{①} + C$

$\boxed{\text{made many mistake } \int \frac{1}{3x^2} = \log_e 3x^2}$

$= -\frac{1}{3x} + \frac{e^{3x}}{3} + C$ ISE

Q3. a) i)



① all 3 points must be clearly labelled.

ii) $m_{AB} = \frac{4-1}{1+2} = \frac{3}{3} = 1$ ①

Eqn AB $y - 4 = 1(x-1)$
$y - 4 = x - 1$ ①

$\therefore\ x - y + 3 = 0$

iii) $d_\perp = \frac{|1\times 2 - 1\times -2 + 3|}{\sqrt{1^2+(-1)^2}}$ ①

C(2, -2)

$= \frac{7}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$ units ①

Q3 continued (iv) $d_{AB} = \sqrt{(1+2)^2 + (4-1)^2}$

$= \sqrt{18}$

$= 3\sqrt{2}$ units (1)

(v) $A = \frac{1}{2} \times 3\sqrt{2} \times \frac{7}{\sqrt{2}}$

$= \frac{21}{2}$

$= 10.5$ units^2 (1)

(b) AP $T_2 - T_1 = T_3 - T_2$

$= T_2 - T_1$ RHS $= T_3 - T_2$

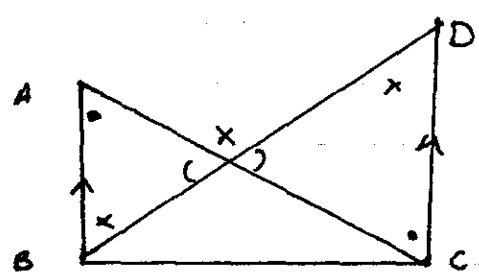
$= \log 4 - \log 2$ $= \log 8 - \log 4$

$= 2\log 2 - \log 2$ $= 3\log 2 - 2\log 2$

$= \log 2$ $= \log 2$

$\therefore$ LHS = RHS (1)

$\therefore$ $\log 2, \log 4, \log 8, \log 16$ is an arithmetic sequence

(c) 

$AB : DC = 2 : 3$

$\therefore \frac{AB}{DC} = \frac{2}{3}$

(i) $B\hat{A}X = X\hat{C}D$ (alternate $\angle$'s =, $AB \parallel DC$)
$A\hat{B}X = X\hat{D}C$ (alternate $\angle$'s =, $AB \parallel DC$)
$A\hat{X}B = D\hat{X}C$ (vertically opposite $\angle$'s =)
(1) for any 2 correct with reasons

$\therefore$ $\Delta AXB ||| \Delta CXD$ (equiangular) (1)

(ii) $BX : XD = 2 : 3$ (corresponding sides in similar Δ's) (1)

$\frac{BX}{XD} = \frac{2}{3}$

$3BX = 2XD$ (1)

$\therefore 9BX^2 = 4XD^2$

Q4 a) i) particle at rest when $v=0$

$\therefore t=2, \quad t=9$ seconds ①

ii) using areas : area below = area above

occurs when $t=4$ seconds ①

iii) Total distance = Sum of areas

$= \frac{1}{2} \times 2 \times 1 + \frac{1}{2} \times 2(2+7)$

$= 10\text{m}$ ①

b) $a=1 \quad r=-x \quad S_\infty = 4 \quad S_\infty = \frac{a}{1-r}$

$4 = \frac{1}{1+x}$ ①

$4x+4=1$

$4x=-3$

$x=\frac{-3}{4}$ ①

c) $6y+11+1 = x^2-2x+1$

$6(y+2) = (x-1)^2$ concave up.

$\therefore$ vertex at $(1, -2)$ ①

$4a=6$

$a=\frac{3}{2}$ focal length ①

$\therefore$ focus $(1, -\frac{1}{2})$ ①

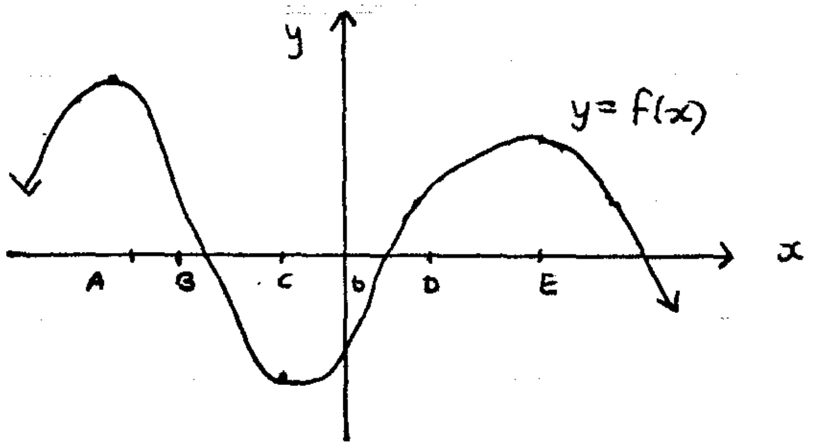
d) i) $\alpha+\beta = \frac{-b}{a} \qquad \alpha+\beta = \frac{3}{2}$

ii) $\alpha\beta = \frac{c}{a} \qquad \alpha\beta = \frac{-6}{2} = -3$

iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta \qquad \alpha^2+\beta^2 = \left(\frac{3}{2}\right)^2 - 2(-3)$ ①

$= 8\frac{1}{4}$ ①

Q5. a)



① shape
① stationary points at A, C, E

b) $f(x) = x^3 - 3x^2 - 9x$
$f'(x) = 3x^2 - 6x - 9$
$f''(x) = 6x - 6$

i) $f'(x) = 0$ $\quad 3(x^2 - 2x - 3) = 0$
$(x-3)(x+1) = 0$
$\therefore x = 3, -1$ ①

* award full marks for correct x values and correct tests (ignore y-value)

When $x = 3$ $\quad f(3) = 27 - 27 - 27 = -27$

①
test $x = 3$ $\quad f''(x) = 18 - 6 = 12 > 0$ $\quad \therefore$ minimum turning point at $(3, -27)$

When $x = -1$ $\quad f(-1) = -1 - 3 + 9 = 5$

①
test $x = -1$ $\quad f''(x) = -6 - 6 = -12 < 0$ $\quad \therefore$ maximum turning point at $(-1, 5)$

ii) $f''(x) = 0$ $\quad 6x - 6 = 0$
$x = 1$

When $x = 1$ $\quad f(1) = 1 - 3 - 9 = -11$.

① test
① point

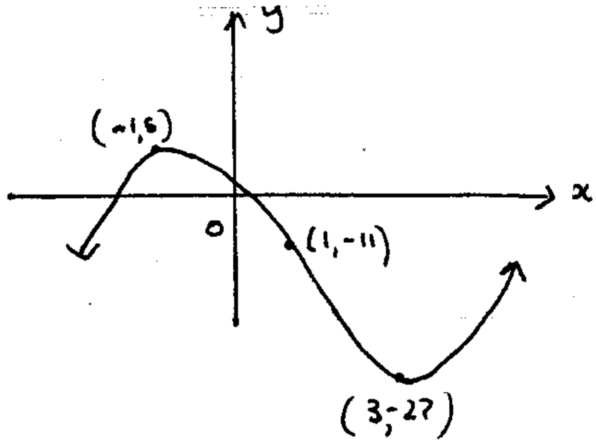
test $x = 1$

x	0	1	2
$f''(x)$	-6	0	6

$\therefore$ change in concavity

$\therefore$ point of inflexion occurs at $(1, -11)$

Q5 continued. iii)



① for correct shape and labelled points (need to include y values)

(iv) $f'(x) < 0 \quad x^2 - 2x - 3 < 0$

$-1 < x < 3$

-1 3

$f''(x) > 0 \quad 6x - 6 > 0$

$x > 1$ ①

$\therefore$ for both to hold: $1 < x < 3$ ①

c) $2\sin^2\theta - 1 = 0$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$ ①

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ ①

AW ① for answers in degrees

ie $\frac{1}{\sqrt{2}}$ only ← Cfm → aw 1 mark for 2 answers (must have all 4 solutions)

Q6. a) $y = A\cos 4x + B\sin 4x$

$\frac{dy}{dx} = -4A\sin 4x + 4B\cos 4x$

$\frac{d^2y}{dx^2} = -16A\cos 4x - 16B\sin 4x$ ①

LHS $= \frac{d^2y}{dx^2} + 16y$

$= -16A\cos 4x - 16B\sin 4x + 16(A\cos 4x + B\sin 4x)$

$= -16A\cos 4x - 16B\sin 4x + 16A\cos 4x + 16B\sin 4x$ ①

$= 0$

$=$ RHS

Q 6 continued

b) i) $R = 20 - \frac{20}{t+1}$

$t = 4 \quad R = 20 - \frac{20}{4+1}$

$R = 16.\ L/day$ 1

ii) as $t \to \infty \quad -\frac{20}{t+1} \to 0$

$\therefore R \to 20$

$\therefore$ rate approaches 20L/day 1

iii) $\int_0^9 20 - \frac{20}{t+1}\, dt = \left[20t - 20\ln(t+1)\right]_0^9$ 1

$= (180 - 20\ln 10) - (0 - 20\ln 1)$

$= 180 - 20\ln 10$ needed exact 1

c) i) $P = 3000e^{-kt}$

$\frac{dP}{dt} = -k\,3000e^{-kt}$ 1

$= -kP$

ii) $2000 = 3000e^{-4k}$ 1

$\frac{2}{3} = e^{-4k}$

$\ln\frac{2}{3} = -4k$

$k = -\frac{1}{4}\ln 2/3$

$= 0.101366277$

$= 0.1014$ 1

iii) $\frac{dP}{dt} = -(-\frac{1}{4}\ln 2/3) \times 3000\, e^{-(-\frac{1}{4}\ln 2/3)\times 7}$

$= -149.5736347$

$= -149.6$ 1

OR using $k = 0.1014$

$= -149.5880795$

$= -149.6$

iv) 10% of 3000 = 300

$300 = 3000e^{-kt}$

$\frac{1}{10} = e^{-kt}$

$t = -\frac{1}{k}\ln\frac{1}{10}$ 1

$t = 22.71549435$

$t = 22.7$ days 1

using k in full

*using $k = 0.1014$

$t = 22.70793977$

Q7. a)

y_0	y_1	y_2	y_3	y_4
0	0.25	0.5	0.75	1.0

$$\int_0^1 10^x\,dx = \frac{0.25}{3}\left[(1+10)+4(10^{0.25}+10^{0.75})+2(10^{0.5})\right] \quad 1$$

$$= 3.910943831$$
$$= 3.911 \quad 1$$

b) i) LHS $= \sin 2x$
$= \sin \frac{2\pi}{8}$
$= \frac{1}{\sqrt{2}}$

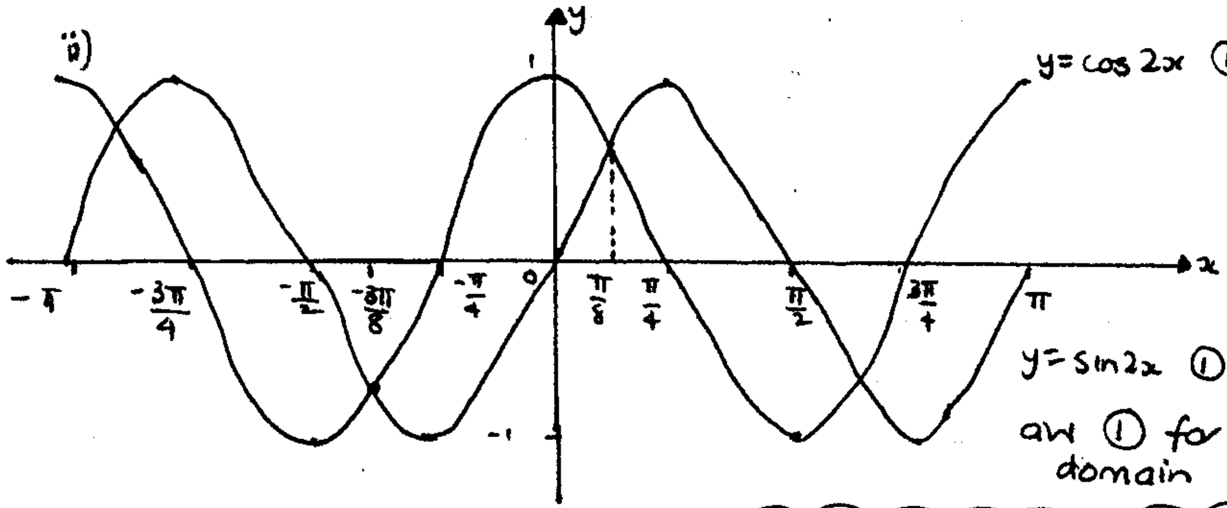
RHS $= \cos 2x$
$= \cos \frac{2\pi}{8}$
$= \frac{1}{\sqrt{2}}$ 1

need both for mark

$\therefore$ LHS = RHS

also $\frac{\sin 2x}{\cos 2x} = 1$
$\tan 2x = 1$
$2x = \frac{\pi}{4}$
$x = \frac{\pi}{8}$

ii)



aw 1 for correct domain

* maximum 1 for amplitude 2 or for $\sin x$, $\cos x$ graphs in the correct domain

(iii) 2 points of intersection
$\therefore \tan 2x = 1 \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$
$\therefore$ 2 solutions

1 accept $-\frac{3\pi}{8}, \frac{\pi}{8}$

(iv) $-\frac{3\pi}{8} \le x \le \frac{\pi}{8}$ 1

Q7 continued

$$y = \sqrt{x} + \frac{1}{\sqrt{x}} \qquad V = \pi\int_a^b y^2\,dx$$

$$y^2 = \left(\sqrt{x} + \tfrac{1}{\sqrt{x}}\right)\left(\sqrt{x} + \tfrac{1}{\sqrt{x}}\right)$$

$$y^2 = x + 2 + \tfrac{1}{x} \quad (1)$$

$$\therefore \text{Volume} = \pi\int_4^{11} x + 2 + \frac{1}{x}\,dx$$

$$= \pi\left[\frac{x^2}{2} + 2x + \ln x\right]_4^{11} \quad (1)$$

$$= \pi\left[\left(\frac{121}{2} + 22 + \ln 11\right) - \left(\frac{16}{2} + 2\times 4 + \ln 4\right)\right] \quad (1)$$

$$= \pi\left[\frac{133}{2} + \ln 11 - \ln 4\right] \quad (1)$$

$$= \pi\left[\frac{133}{2} + \ln\frac{11}{4}\right]$$

Q8 a) (i) $y = 4 - x^2$

test $(0,0)$ $\quad 0 \leq 4 - 0^2$ True $\qquad$ poorly attempted

$\therefore y \leq 4 - x^2$ (1)

$y = x^2 - 2x$

Test $(1,0)$ $\quad 0 \geq 1 - 2$

$\therefore y \geq x^2 - 2x$ (1)

ii)
$$4 - x^2 = x^2 - 2x$$
$$2x^2 - 2x - 4 = 0$$
$$2(x^2 - x - 2) = 0$$
$$(x-2)(x+1) = 0 \quad (1)$$

$\therefore x = 2, -1$ as required

iii)
$$\text{Area} = \int_{-1}^{2} 4 - x^2 - x^2 + 2x\,dx$$

$$= \int_{-1}^{2} 4 + 2x - 2x^2\,dx$$

$$= \left[4x + x^2 - \frac{2x^3}{3}\right]_{-1}^{2} \quad (1)$$

$$= \left[\left(8 + 4 - \frac{16}{3}\right) - \left(-4 + 1 + \frac{2}{3}\right)\right] = 9 \text{ units}^2 \quad (1)$$

Q8 continued

b) i) $t=0$ $\quad x = 0 + \cos 0$

$x = 1$ m ① (ie 1m to right of origin)

ii) Particle at rest $v=0$ $\quad x = t + \cos t$

$\frac{dx}{dt} = 1 - \sin t$ ①

$1 - \sin t = 0$

$\sin t = 1$

$t = \frac{\pi}{2}$ seconds ①

When $t = \frac{\pi}{2}$ sec $\quad x = \frac{\pi}{2} + \cos\frac{\pi}{2}$

$= \frac{\pi}{2}$ metres ①

$(= 1.570796327)$

iii) $t = \frac{5\pi}{2}$ seconds (ie $\frac{\pi}{2} + 2\pi$) ①

iv) $\frac{d^2x}{dt^2} = -\cos t$ ①

When $t = \frac{\pi}{6}$ $\quad a = -\cos\frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$ ms^{-2} ① Accept -0.866025403 -0.866

Q9. a) $\frac{\operatorname{cosec} A \sec A}{\tan A} = \frac{1}{\sin A \cancel{\cos A}} \times \frac{\cancel{\cos A}}{\sin A}$

$= \frac{1}{\sin^2 A}$

$= \operatorname{cosec}^2 A$ ①

b) Depth 3, 5, 7, ...

Cost 260, 280, 300, ...

i) Depth $a=3$ $d=2$ $T_n = 27$

$27 = 3 + (n-1) \times 2$

$n-1 = 12$ $\quad \therefore n = 13$ ①

Q 9 continued

Cost $a = 260$ $d = 20$ $n = 13$

$$T_{13} = 260 + (13-1) \times 20 \quad 1$$
$$= \$500$$

(ii) $$S_{13} = \frac{13}{2}\left(2 \times 260 + (13-1) \times 20\right)$$
$$= \$4940 \quad 1$$

(iii) $S_n = 12500$

$$12500 = \frac{n}{2}\left(520 + (n-1) \times 20\right)$$
$$25000 = n(520 + 20n - 20)$$
$$25000 = 500n + 20n^2$$
$$n^2 + 25n - 1250 = 0$$
$$(n-25)(n+50) = 0$$
$$n = 25, -50 \quad \text{but } n > 0$$
$$\therefore n = 25. \quad 1$$

Depth $$T_{25} = 3 + (25-1) \times 2$$
$$= 51m \quad 1$$

(c) i) $$A_1 = P(1.005) - 3500 \quad 1$$

P
$r = 6\%$ p.a $= 0.5\%$ p.m
$n = 3$ months
repayments $= 3500$

ii) $$A_2 = A_1(1.005) - 3500$$
$$= P(1.005)^2 - 3500(1 + 1.005) \quad 1$$
$$A_3 = A_2(1.005) - 3500$$
$$= P(1.005)^3 - 3500(1.005)^2 - 3500(1.005) - 3500 \quad 1$$
$$\therefore A_3 = P(1.005)^3 - 3500(1 + 1.005 + 1.005^2)$$

iii) $$A_4 = A_3(1.0075) - 4200$$

$r = 9\%$ p.a $= 0.75\%$ p.m
$n = 57$ months
repayments $= 4200$

$$A_5 = A_4(1.0075) - 4200$$
$$= (A_3(1.0075) - 4200)1.0075 - 4200$$
$$= A_3(1.0075)^2 - 4200(1.0075) - 4200$$
$$= A_3(1.0075)^2 - 4200(1.0075 + 1)$$
$$\therefore A_6 = A_3(1.0075)^3 - 4200(1 + 1.0075 + 1.0075^2)$$
$$\vdots$$

Q9 continued ①

(c) iii) $A_{60} = A_3 (1.0075)^{57} - 4200\underbrace{(1 + 1.0075 + 1.0075^2 + \ldots + 1.0075^{56})}_{\text{geometric series}}$

geometric series $a=1$ $r=1.0075$ $n=57$

but loan is repaid $\therefore A_{60} = 0$

$$0 = A_3 (1.0075)^{57} - 4200 \times \frac{1\,(1.0075^{57} - 1)}{1.0075 - 1}$$

$$A_3 (1.0075)^{57} = \frac{4200\,(1.0075^{57} - 1)}{1.0075 - 1}$$

from (ii) use A_3 ①

$$P\left[(1.005)^3 - 3500(1 + 1.005 + 1.005^2)\right](1.0075)^{57} = \frac{4200(1.0075^{57} - 1)}{1.0075 - 1}$$

$$\therefore P = \left[\frac{4200\,(1.0075^{57} - 1)}{(1.0075 - 1)(1.0075)^{57}} + 3500\,(1 + 1.005 + 1.005^2)\right] \div (1.005)^3$$

$P = 201731.5044$ ① (ISE)

$P = \$201\,731.50$

$P = \$201\,732$

Q10. a) area of segment $= \frac{1}{2}\theta(R^2 - r^2)$

$$5\pi = \frac{1}{2} \times \frac{\pi}{6}(R^2 - 2^2)$$

$$60 = R^2 - 4$$

$$R^2 = 64$$

$R = 8$ as $R > 0$ ①

$\therefore OC = 8$ cm

$\therefore AC = 8 - 2 = 6$ cm. ① mark only awarded for correct answer

Q10 continued.

b) i) distance AP (cosine rule)

$$AP^2 = r^2 + (r+a)^2 - 2r(r+a)\cos\phi$$
$$AP = \sqrt{r^2 + (r+a)^2 - 2r(r+a)\cos\phi}$$

$$\text{time taken} = \frac{AP}{v_1} = \frac{\sqrt{r^2+(r+a)^2-2r(r+a)\cos\phi}}{v_1} \quad (1)$$

distance PC (arc length)

$$l = r\theta = r \times (\pi - \phi)$$
$$(\hat{COP})$$

$$\text{time taken} = \frac{PC}{v_2} = \frac{r(\pi-\phi)}{v_2} \quad (1)$$

$$\text{total time } (T) = \frac{AP}{v_1} + \frac{PC}{v_2}$$
$$= \frac{\sqrt{r^2+(r+a)^2-2r(r+a)\cos\phi}}{v_1} + \frac{r(\pi-\phi)}{v_2} \quad (1)$$

ii)
$$\frac{dT}{d\phi} = \cancel{\tfrac{1}{2}}\frac{\left(r^2+(r+a)^2-2r(r+a)\cos\phi\right)^{-\frac{1}{2}}}{v_1} \times \cancel{2}r(r+a)\sin\phi - \frac{r}{v_2}$$
$$= \frac{r(r+a)\sin\phi}{v_1\sqrt{r^2+(r+a)^2-2r(r+a)\cos\phi}} - \frac{r}{v_2} \quad (1)$$

iii) using sine rule : $\frac{r}{\sin\theta} = \frac{AP}{\sin\phi} \Rightarrow \frac{\sin\phi}{AP} = \frac{\sin\theta}{r}$

$$\frac{dT}{d\phi} = \frac{r(r+a)\sin\phi}{v_1 AP} - \frac{r}{v_2} \quad \text{(using (i))} \quad (1)$$
$$= \frac{r(r+a)}{v_1} \times \frac{\sin\theta}{r} - \frac{r}{v_2} \quad (1)$$
$$= \frac{r+a}{v_1}\left[\sin\theta - \frac{rv_1}{v_2(r+a)}\right]$$

Q10 continued

(c) (iv) When P is at B $\theta = 0$

$$\frac{dT}{d\phi} = \frac{r+a}{v_1}\left[\sin 0 - \frac{rv_1}{v_2(r+a)}\right]$$

$$= -\frac{r}{v_2} < 0 \quad \text{since } v_2 > 0 \qquad (1)$$

When P is at X $\quad A\hat{X}O = 90^\circ \quad \therefore \sin\theta = \frac{r}{r+a}$

$$\frac{dT}{d\phi} = \frac{r+a}{v_1}\left[\frac{r}{r+a} - \frac{rv_1}{v_2(r+a)}\right]$$

$$= \frac{r}{v_1} - \frac{r}{v_2}$$

$$= \frac{r(v_2 - v_1)}{v_1 v_2} > 0 \quad \text{since } r > 0,\ v_2 > v_1 > 0 \qquad (1)$$

(v). minimum $\Rightarrow$ stationary point $\Rightarrow \frac{dT}{d\phi} = 0$

$$0 = \frac{r+a}{v_1}\left[\sin\theta - \frac{rv_1}{v_2(r+a)}\right]$$

$$\therefore \sin\theta = \frac{rv_1}{v_2(r+a)}$$

using (iv) Test

$\sin\theta$ / $\frac{dT}{d\phi}$	at B	at P	at X
	< 0	$= 0$	> 0
	$-$	0	$+$

(1)

min

$\therefore$ minimum occurs when $\sin\theta = \frac{rv_1}{v_2(r+a)}$ (1)